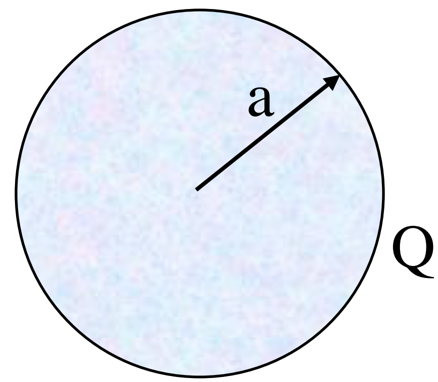
Problem 24.37

Consider a metal sphere of radius *a* and charge Q.

Just outside,
$$E = k\frac{Q}{a^2}$$
$$= \frac{1}{4\pi\varepsilon_o}\frac{Q}{a^2}$$



What is this in terms of $\sigma\left(=\frac{Q}{4\pi a^2}\right)$?

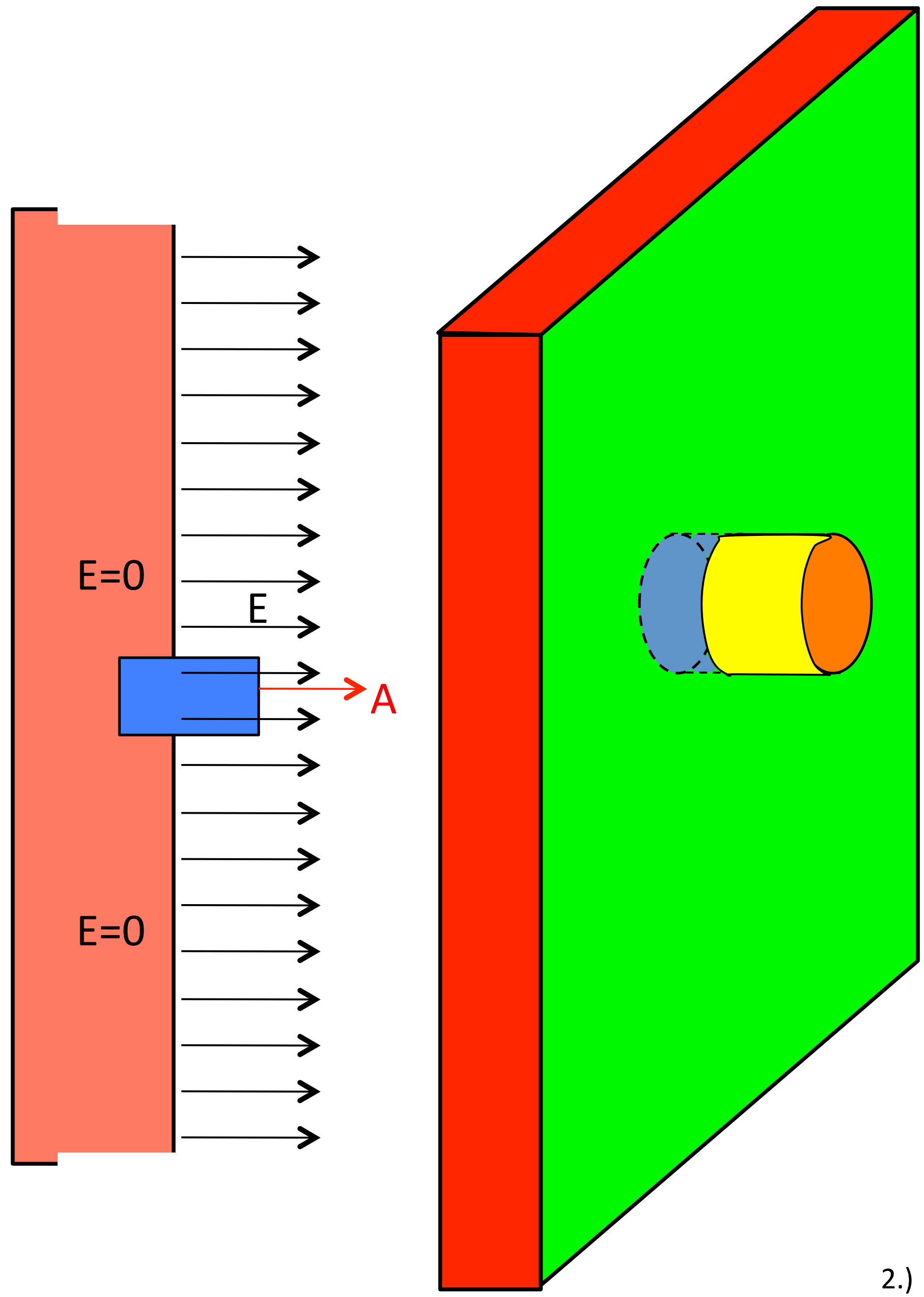
Manipulating: $E = \left(\frac{Q}{4\pi a^2}\right)\frac{1}{\varepsilon_o} = \frac{\sigma}{\varepsilon_o}$

The other alternative is to simply derive E using a surface charge density function, and see what you get:

Consider a flat, metallic surface: On one side, where the charge is located, an electric field exists directed outward from the surface. On the other side (inside the conductor), the field is zero.

Placing a Gaussian plug as shown in the sketch, we have electric field lines passing through the plug's end (area "A") on one side.

The "charge enclosed" inside the Gaussian surface will equal the *charge per unit area* σ times the area "A."



In short, as long as you are so close to the sphere that it looks like a flat surface, Gauss's Law maintains that:

$$\int E \bullet dS = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\Rightarrow \quad EA = \frac{\sigma A}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_o}$$